

Repeat Accumulate Based Designs for LDPC Codes on Fading Channels

Andre G. D. Uchoa, Cornelius Healy and Rodrigo C. de Lamare

University of York,
Heslington York, YO10 5DD, UK
{agdu500, cth503, rcd1500}@york.ac.uk

Abstract—Irregular repeat-accumulate Root-Check LDPC codes based on Progressive Edge Growth (PEG) techniques for block-fading channels are proposed. The proposed Root-Check LDPC codes are both suitable for channels under $F = 2, 3$ independent fading per codeword and for fast fading channels. An IRA(A) Root-Check structure is devised for $F = 2, 3$ independent fading. The performance of the new codes is investigated in terms of the Frame Error Rate (FER). Numerical results show that the IRAA LDPC codes constructed by the proposed algorithm outperform by about 1dB the existing IRA Root-Check LDPC codes under fast-fading channels.

Index Terms—LDPC, Root-Check, PEG, Repeat Accumulate

I. INTRODUCTION

Due to multi-path propagation and mobility, wireless systems are characterized by time-varying channels with fluctuating signal strength. In applications subject to delay constraints and slowly-varying channels, only limited independent fading realizations are experienced. In such conditions also known as non-ergodic scenarios, the channel capacity is zero since there is an irreducible probability, termed outage probability [1], that the transmitted data rate is not supported by the channel. The case of interest in this work is the block-fading type [2]. A simple and useful model that captures the essential characteristics of non-ergodic channels is the block-fading channel [2]. It is especially important in wireless communications with slow time-frequency hopping (e.g., cellular networks and wireless local area networks) or multi-carrier modulation using Orthogonal Frequency Division Multiplexing (OFDM) [3]. Codes designed for block-fading channels are expected to achieve the limited channel diversity and to offer good coding gains.

In [3] the authors proposed a family of LDPC codes called Root-Check for block-fading channels. Root-check codes are able to achieve the maximum diversity of a block-fading channel and have a performance near the limit of outage when decoded using the Sum Product Algorithm (SPA). Root-check codes are always designed with code rate $R = 1/F$, since the Singleton bound determines that this is the highest code rate possible to obtain the maximum diversity order [3]. Li and Salehi [4] proposed the construction of structured Root-Check LDPC codes via circulating matrices, i.e., Quasi-Cyclic LDPC codes (QC-LDPC). In [4] the authors have shown that the QC-LDPC codes are as good as the randomly generated Root-Check LDPC codes on block-fading channels. It is known that the girth, the length of the shortest cycle in the graph of this code has a significant effect on the performance of the code. Among the algorithms capable of producing high performance LDPC codes for short to medium lengths is the Progressive Edge Growth (PEG) algorithm [5].

In order to improve the girth of the Root-Check LDPC codes the PEG-Based Root-Check LDPC codes [6]–[8] which are designed in a PEG based technique [9] have been developed. The proposed PEG-Based Root-Check LDPC codes presented in [6]–[8] outperformed other LDPC codes based on the Root-Check structure. The best result presented by the works in [6]–[8] reinforces that a Root-Check LDPC code generated with an algorithm based on PEG produces a better performance than that of a standard design.

The accumulator-based codes that were invented first are the so-called repeat-accumulate (RA) codes [10]. Despite their simple structure, they were shown to provide good performance and, more importantly, they paved a path toward the design of efficiently encodable LDPC codes. The contribution of this paper is to present a PEG-based algorithm to design IRA Based LDPC codes with Root-Check properties for block-fading channel with $F = 2, 3$ fading per coded word. The key points in using IRA Based Root-Check LDPC codes are the simplicity in designing such codes, faster encoding than conventional LDPC methods and flexibility in terms of rate compatibility as provided by intentional puncturing strategies [10]. A strategy that imposes irregular repeat-accumulate and Root-Check constraints on a PEG-based algorithm is devised. The codes generated by the proposed algorithm can achieve a very good performance in terms of Frame Error Rate (FER) with respect to the theoretical limit. The proposed design can save up to 1dB in terms of signal to noise ratio (SNR) to achieve the same FER when compared to other codes under fast fading channels. For the case of block-fading channels, the codes achieve a comparable performance in terms of FER as the works in [6]–[8].

The rest of this paper is organized as follows. In Section II we define the system model, while in Section III we present the structure of a RA-Root-Check LDPC codes for $F = 2, 3$ fading. In Section IV we introduce the proposed PEG-based algorithm. Section V presents numerical results, while Section V concludes the paper.

II. SYSTEM MODEL

Consider a block fading channel, where F is the number of independent fading blocks per codeword of length N . Following [4], the t -th received symbol is given by:

$$r_t = h_f s_t + n_{g_t}, \quad (1)$$

where $t = \{1, 2, \dots, N\}$, $f = \{1, 2, \dots, F\}$, f and t are related by $f = \lceil F \frac{t}{N} \rceil$, where $\lceil \phi \rceil$ returns the smallest integer not smaller than ϕ , h_f is the real Rayleigh fading coefficient of the f -th block, s_t is the transmitted signal, and n_{g_t} is additive white Gaussian noise with zero mean and variance $N_0/2$. In

the case of fast fading we assume that each received symbol r_t will be under a distinct fading coefficient, which means $F = N$. In this paper, we assume that the transmitted symbols s_t are binary phase shift keying (BPSK) modulated. We assume that the receiver has perfect channel state information, and that the SNR is defined as E_b/N_0 , where E_b is the energy per information bit. The information transmission rate is $R = K/N$, where K is the number of information bits per codeword of length N . For the case of a block-fading channel, we consider $R = 1/F$, since then it is possible to design a practical diversity achieving code [4]. The performance of a communication system in a non-ergodic block-fading channel can be investigated by means of the outage probability [4], which is defined as:

$$P_{out} = \mathcal{P}(I < R), \quad (2)$$

where $\mathcal{P}(\phi)$ is the probability of event ϕ . The mutual information I_G , for Gaussian channel inputs is [4]:

$$I_G = \frac{1}{F} \sum_{f=1}^F \frac{1}{2} \log_2 \left(1 + 2R \frac{E_b}{N_0} h_f^2 \right), \quad (3)$$

so that an outage occurs when the average accumulated mutual information among blocks is smaller than the attempted information transmission rate.

III. RA BASED LDPC CODES DESIGN

A repeat-accumulate (RA) code consists of a serial concatenation, through an interleaver, of a single rate $1/q$ repetition code with an accumulator having transfer function $\frac{1}{1+D}$, where q is the number of repetitions for each group of K information bits. Fig. 1 shows a typical repeat-accumulate code block diagram. The implementation of the transfer function $\frac{1}{1+D}$ is identical to an accumulator, although the accumulator value can be only 0 or 1 since the operations are over the binary field [10]. As discussed in [10], to ensure a large minimum Hamming distance, the interleaver should be designed so that consecutive 1s at its input are widely separated at its output. The RA based codes discussed throughout this paper will be systematic. The main limitation of RA codes are the code rate, which cannot be higher than $1/2$. Also, these codes are not capacity-approaching. Therefore, we will be discussing in the next subsections some enhancements to RA codes which permit operation closer to the block-fading outage probability.

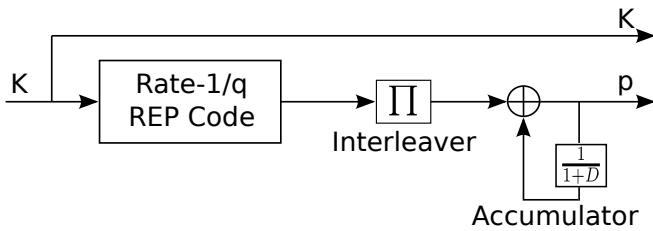


Figure 1. A systematic repeat-accumulate code block diagram, where K is the number of information bits and p denotes the parity bits.

A. IRA Root-Check Design

Irregular repeat-accumulate (IRA) codes generalize the concept of RA codes by changing the repetition rate for

each group of K information bits and performing a linear combination of the repeated bits which are sent through the accumulator. Furthermore, IRA codes are typically systematic. IRA codes allow flexibility in the choice of the repetition rate for each information bit so that high-rate codes may be designed. And, their irregularity allows operation closer to the capacity limit [10].

The Parity Check Matrix (PCM) for systematic RA and IRA codes has the form $\mathbf{H} = [\mathbf{H}_u \mathbf{H}_p]$, where \mathbf{H}_p is a square dual-diagonal matrix given by

$$\mathbf{H}_p = \begin{bmatrix} 1 & & & & \\ 1 & 1 & & & \\ & \ddots & \ddots & & \\ & & 1 & 1 & \\ & & & 1 & 1 \end{bmatrix}. \quad (4)$$

For RA codes, \mathbf{H}_u is a regular matrix having column weight q and row weight 1. For IRA codes, \mathbf{H}_u has irregular columns and rows weights. The Generator Matrix (GM) can be obtained as $\mathbf{G} = [\mathbf{I} \mathbf{H}_u^T \mathbf{H}_p^T]$, where \mathbf{I} is an identity matrix of size $K \times K$. In matrix notation \mathbf{H}_p^{-T} can be described as

$$\mathbf{H}_p^{-T} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ & 1 & 1 & \cdots & 1 \\ & & \ddots & & \vdots \\ & & & 1 & 1 \\ & & & & 1 \end{bmatrix}. \quad (5)$$

1) *IRA Root-Check Rate $\frac{1}{2}$* : To design a Root-Check with an IRA structure we have imposed some constraints in terms of parity check matrix to guarantee the Root-Check properties. Following the notation adopted in [6], for the case of a systematic Rate $1/2$ with $F = 2$, the PCM must be like

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} & \mathbf{H}_2 & \mathbf{0} & \mathbf{H}_3 \\ \mathbf{H}_2 & \mathbf{I} & \mathbf{H}_3 & \mathbf{0} \end{bmatrix}, \quad (6)$$

where \mathbf{H}_2 and \mathbf{H}_3 are $\frac{N}{2} \times \frac{N}{2}$ sub-matrices with Hamming weight two and three respectively, while $\mathbf{0}$ is a null sub-matrix. Therefore, to impose the RA structure and Root-Check properties the PCM of an IRA Root-Check is

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} & \mathbf{H}_2 & \mathbf{0} & \mathbf{H}_p \\ \mathbf{H}_2 & \mathbf{I} & \mathbf{H}_p & \mathbf{0} \end{bmatrix}, \quad (7)$$

where \mathbf{H}_p is a dual diagonal matrix with size $\frac{N}{2} \times \frac{N}{2}$. By doing this, we are able to achieve the diversity of the channel and the same performance in terms of FER as the codes designed in [6], [7].

2) *IRA Root-Check Rate $\frac{1}{3}$* : For the case of Rate $1/3$ with $F = 3$, we follow a similar structure to the one adopted in [3], [8]. However, we have made some modifications on the accumulator to approach the outage probability. The accumulator used in this case has a transfer function $\frac{1}{1+D+D^{\frac{N}{9}}}$ as suggested in [11]. As a result, \mathbf{H}_p must be redefined as

$$\mathbf{H}_p = \begin{bmatrix} \mathbf{H}_{p1} \\ \mathbf{H}_{p2} \end{bmatrix}, \quad (8)$$

$$\mathbf{H}_{p1} = \begin{bmatrix} 1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 1 & 1 & 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (9)$$

$$\mathbf{H}_{p2} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & 1 & 1 & 0 & 0 \\ \vdots & 0 & \ddots & 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}, \quad (10)$$

where \mathbf{H}_{p1} and \mathbf{H}_{p2} are sub-matrices with dimensions $\frac{N}{9} \times \frac{2N}{9}$. Now, we can show the final PCM $H = [\mathbf{H}_u | \mathbf{H}_p]$ for the case of an IRA Root-Check Rate $1/3$ as

$$\mathbf{H} = \left[\begin{array}{ccc|ccc} \mathbf{I} & \mathbf{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{H}_{p2} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{H}_{p1} \\ \mathbf{H}_1 & \mathbf{I} & \mathbf{0} & \mathbf{H}_{p1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{H}_1 & \mathbf{0} & \mathbf{0} & \mathbf{H}_{p2} \\ \mathbf{H}_1 & \mathbf{0} & \mathbf{I} & \mathbf{H}_{p2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1 & \mathbf{I} & \mathbf{0} & \mathbf{H}_{p1} & \mathbf{0} \end{array} \right], \quad (11)$$

where \mathbf{H}_1 and \mathbf{I} are sub-matrices with dimensions $\frac{N}{9} \times \frac{N}{9}$ and \mathbf{H}_1 is a sub-matrix with Hamming weight equal to 1. The null sub-matrices $\mathbf{0}$ in the right hand side of (11) have dimensions $\frac{N}{9} \times \frac{2N}{9}$ while in the left hand side the dimensions are $\frac{N}{9} \times \frac{N}{9}$. The reason why we split \mathbf{H}_p into two sub-matrices \mathbf{H}_{p1} and \mathbf{H}_{p2} is to guarantee the full rank and full diversity properties stated in [3].

B. IRAA Root-Check Design

The general structure of an Irregular Repeat-Accumulate and Accumulate (IRAA) encoder can be seen in Fig. 2. In this figure, we see an extra parity bits which are termed b and the normal parity bits p . The b parity bits can be punctured to obtain a higher code rate. For instance, in general an IRAA code is rate $1/3$ without puncturing, while puncturing b it can be obtained a code with rate $1/2$.

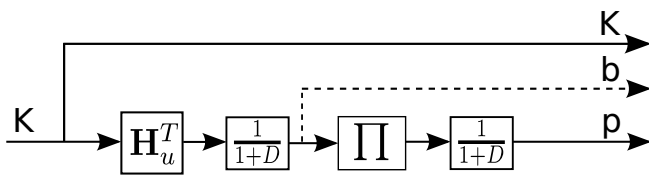


Figure 2. An systematic irregular repeat-accumulate and accumulate code block diagram. Where K are the information bits, b and p are the parity bits.

The PCM of an IRAA LDPC code can be represented by

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_u & \mathbf{H}_p & \mathbf{0} \\ \mathbf{0} & \prod_1 & \mathbf{H}_p \end{bmatrix}, \quad (12)$$

where \prod_1 must be a sub-matrix with rows and columns with Hamming weight one.

In order to obtain IRAA Root-Check LDPC codes some constraints must be imposed on the standard IRAA design. We have noticed that the IRAA Root-Check LDPC codes led to a more flexible rate compatible code and a better performance under fast fading channels. This will be illustrated by simulations later on in Section V.

1) *IRAA Root-Check Rate $\frac{1}{2}$* : We applied the Root-Check structure from (7) in (12) to obtain the following PCM for rate $1/2$

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} & \mathbf{H}_2 & \mathbf{0} & \mathbf{H}_p & \mathbf{0} & \mathbf{0} \\ \mathbf{H}_2 & \mathbf{I} & \mathbf{H}_p & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \prod_1 & \mathbf{0} & \mathbf{H}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H}_p & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad (13)$$

where \mathbf{I} , \mathbf{H}_2 , \mathbf{H}_2 and $\mathbf{0}$ are all $\frac{N}{9} \times \frac{N}{9}$ in dimension, while \prod_1 is $\frac{N}{3} \times \frac{N}{3}$. The key point to guarantee the full diversity property is the puncturing procedure. Instead of puncturing b parity bits we have punctured p . The reason why puncturing p instead of b guarantees the full diversity is due to the fact that the Root-Check structure of the code is kept unchanged. For the case of fast fading we have punctured in the same manner as for the case of block-fading channels.

2) *IRAA Root-Check Rate $\frac{1}{3}$* : For the case of rate $1/3$ we considered the design done in (11) and we apply the constraints in (12) to obtain the following PCM

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_u & \mathbf{H}_p & \mathbf{0} \\ \mathbf{0} & \prod_1 & \mathbf{H}_p \end{bmatrix}. \quad (14)$$

It must be noted that without puncturing the code rate is $1/5$.

IV. PROPOSED DESIGN ALGORITHM

Here, we introduce some definitions and a specific notation. Then, we present the pseudo-code of our proposed algorithm. In this work, the scenarios of a block-fading channel with $F = 2$, $F = 3$ and a fast fading channel are considered. In extending to a greater number of fadings, $F > 4$, the general structure presented is maintained. The LDPC code in systematic form is specified by its sparse PCM $\mathbf{H} = [\mathbf{A} | \mathbf{B}]$, where \mathbf{A} is a matrix of size $M \times K$, and \mathbf{B} is an $M \times M$ matrix. The generator matrix (GM) for the code is $\mathbf{G} = [\mathbf{I} | (\mathbf{B}^{-1} \mathbf{A})^T]$, \mathbf{I} is an identity matrix of size $K \times K$.

The variable node degree sequence D_s is defined as the set of column weights of the designed \mathbf{H} , and is prescribed by the variable node degree distribution $\lambda(x)$ as described in [12]. Moreover, D_s is arranged in non-decreasing order. The proposed algorithm, called IRA-PEG Root-Check, constructs \mathbf{H} by operating progressively on variable nodes to place the edges required by D_s . The Variable Node (VN) of interest is labelled v_j and the candidate check nodes are individually referred to as c_i . The IRA-PEG Root-Check algorithm chooses a check node c_i to connect to the variable node of interest v_j by expanding a constrained sub-graph from v_j up to maximum depth l . The set of check nodes found in this sub-graph is denoted $N_{v_j}^l$ while the set of check nodes of interest, those not currently found in the sub-graph, are denoted $\overline{N_{v_j}^l}$.

A. Pseudo-code for the IRA-PEG Root-Check Algorithm

Initialization: A matrix of size $M \times N$ is created with the identity matrices \mathbf{I} and parity matrices \mathbf{H}_p in the positions shown in (7), (11), (13), (14) and zeros in all other positions. We define the indicator vectors $\mathbf{z}_1, \dots, \mathbf{z}_F$ for the cases $R = \frac{1}{2}$, $R = \frac{1}{3}$ respectively as:

$$\begin{aligned} \mathbf{z}_1 &= [\mathbf{0}_{1 \times \gamma}, \mathbf{1}_{1 \times \gamma}]^T, \\ \mathbf{z}_2 &= [\mathbf{1}_{1 \times \gamma}, \mathbf{0}_{1 \times \gamma}]^T, \end{aligned} \quad (15)$$

$$\begin{aligned} \mathbf{z}_1 &= [\mathbf{0}_{1 \times 2\chi}, \mathbf{1}_{1 \times \chi}, \mathbf{0}_{1 \times \chi}, \mathbf{1}_{1 \times \chi}, \mathbf{0}_{1 \times \chi}]^T, \\ \mathbf{z}_2 &= [\mathbf{1}_{1 \times \chi}, \mathbf{0}_{1 \times 4\chi}, \mathbf{1}_{1 \times \chi}]^T, \\ \mathbf{z}_3 &= [\mathbf{0}_{1 \times \chi}, \mathbf{1}_{1 \times \chi}, \mathbf{0}_{1 \times \chi}, \mathbf{1}_{1 \times \chi}, \mathbf{0}_{1 \times 2\chi}]^T, \end{aligned} \quad (16)$$

where $\gamma = \frac{N}{2}$ for the case of IRA, while for IRAA design $\gamma = \frac{N}{4}$. And, $\chi = \frac{N}{9}$ for the case of IRA, while for IRAA design $\chi = \frac{N}{15}$. In addition, for rate $R = \frac{1}{2}$ under IRAA design $\mathbf{z}_i = [\mathbf{z}_i, \mathbf{0}_{4 \times \gamma}]$, while for rate $R = \frac{1}{3}$ under IRAA design $\mathbf{z}_i = [\mathbf{z}_i, \mathbf{0}_{6 \times \chi}]$.

These indicator vectors are modelled on that of the original PEG algorithm [5], indicating sub-matrices for which placement is permitted, thus imposing the form of (7), (11), (13), (14). The degree sequence as defined for LDPC codes must be altered to take into account the structure imposed by Root-Check codes, namely the identity matrices \mathbf{I} and the parity matrices \mathbf{H}_p , of (7), (11), (13) and (14). The pseudo-code for our proposed IRA-PEG Root-Check algorithm is detailed in Algorithm 1, where the indicator vector \mathbf{z}_i is taken from (15), (16) for constructing codes of rate $R = \frac{1}{2}$, $R = \frac{1}{3}$ respectively.

Algorithm 1 IRA-PEG Root-Check Algorithm

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1. for  $j = 1 : K$  do
2.   for  $k = 0 : D_s(j) - 1$  do
3.     if  $j \geq \frac{N}{F}$  &  $k == 0$  then
4.       Place edge at random among minimum weight
         CNs of the CN set permitted by the indicator  $\mathbf{z}_j$ .
5.     else
6.       Expand the PEG tree from the  $\frac{(j-1) \cdot N}{F^2}$ -th VN to
         depth  $l$  such that the tree contains all CNs allowed
         by the indicator vector or the number of nodes in
         the tree does not increase with an expansion to the
          $(l+1)$ -th level.
7.       Place edge connecting the  $\frac{(j-1) \cdot N}{F^2}$ -th VN to a CN
         chosen randomly from the set of minimum weight
         nodes which were added to the sub-tree at the last
         tree expansion.
8.     end if
9.   end for
10. end for

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V. SIMULATIONS RESULTS

The performance of the proposed IRA and IRAA PEG Root-Check LDPC codes when used in a Rayleigh block-fading channel with $F = 2$ and $F = 3$ independent fading blocks is analysed. Moreover, we considered the performance of such

codes under fast fading channels. All LDPC codes simulated have the same degree distribution under the systematic parity of PCM. Standard SPA algorithm is employed at the decoder with a maximum of 20 iterations. Following [3], [4], a maximum of 20 iterations are enough to obtain a good performance in terms of FER for fading channels. The Gaussian outage limit in (3) is drawn in dashed line in each figure for reference. Our proposed IRA-PEG Root-Check codes have a minimum girth of 12.

A. Performance for rate $R = \frac{1}{2}$

In Fig. 3 it is compared the FER performance among the proposed IRA-PEG Root-Check LDPC, IRAA-PEG Root-Check LDPC, PEG Root-Check LDPC from [7], QC Root-Check LDPC from [4] and PEG based LDPC [5] codes, all for $R = \frac{1}{2}$. The codeword length is $L = 1200$ bits. From the results, it can be noted that the proposed IRA(A)-PEG Root-Check LDPC codes perform as well as the PEG Root-Check LDPC code design. The proposed IRA(A)-PEG Root-Check codes bring the key advantage of having less computational encoding complexity than the other methods. Moreover, note that all Root-Check-based codes are able to achieve the full diversity order of the channel, while the non-root-check PEG LDPC codes fail to achieve full diversity. ANDRÉ, WHAT ABOUT THE COMPLEXITY IN COMPARISON WITH QC-BASED DESIGN? PLEASE COMMENT ON THAT.

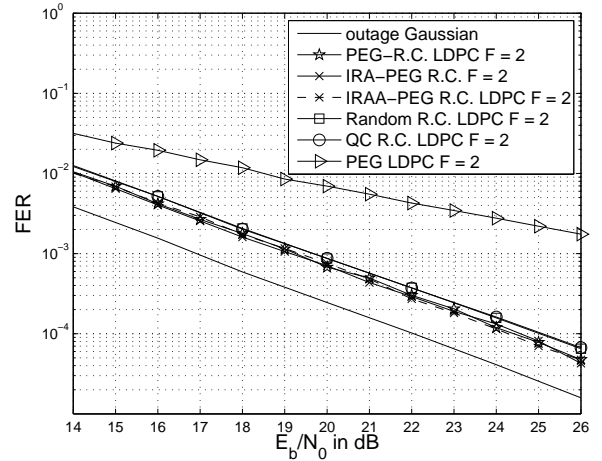


Figure 3. FER performance for the IRA-PEG Root-Check LDPC, IRAA-PEG Root-Check LDPC, PEG Root-Check LDPC, QC Root-Check LDPC, and PEG based LDPC codes over a block-fading channel with $F = 2$ and $L = 1200$ bits. The maximum number of iterations is 20.

B. Performance for rate $R = \frac{1}{3}$

In Fig. 4 it is compared the FER performance among the proposed IRA-PEG Root-Check LDPC, IRAA-PEG Root-Check LDPC, QC-PEG Root-Check LDPC and QC-PEG LDPC, all for rate $R = \frac{1}{3}$. The codeword length is $L = 900$ bits. Similar to the results for $R = \frac{1}{2}$, it can be noted that the proposed IRA(A)-PEG Root-Check LDPC codes perform as well as the PEG Root-Check LDPC code design.

C. Performance for fast fading channels

In Fig. 5 it is compared the FER performance between the proposed IRA-PEG Root-Check LDPC and IRAA-PEG

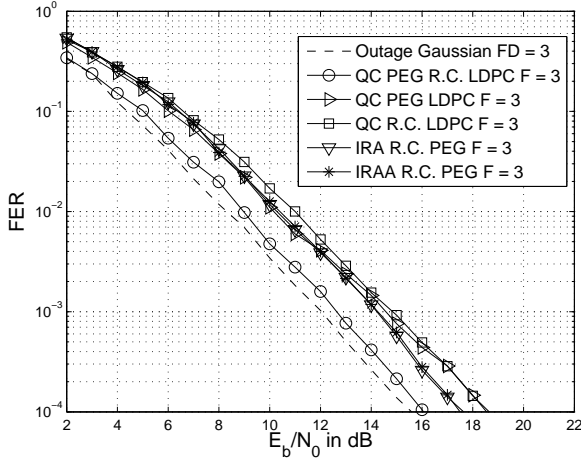


Figure 4. FER performance for the IRA-PEG Root-Check LDPC, IRAA-PEG Root-Check LDPC, QC-PEG Root-Check LDPC and QC-PEG LDPC codes over a block-fading channel with $F = 3$ and $L = 900$ bits. The maximum number of iterations is 20.

Root-Check LDPC codes for the case of fast fading and over different code rates. From the legends in Fig. 5, "PUNC." it means a punctured version of IRAA design to obtain the same code rate as its counterpart IRA code design. From the results, we can see that the IRAA-PEG Root-Check design with rate 1/2 is outperformed by the IRA-PEG Root-Check design by about 0.75dB on average for the same FER. Moreover, the non-punctured version of the IRAA-PEG Root-Check design with rate 1/3 was outperformed by the , the IRA-PEG Root-Check design with rate 1/3 by an average margin of 1.25dB. Nevertheless, we can see that for the case of IRAA-PEG Root-Check with rate 1/3 punctured outperforms IRA-PEG Root-Check design with rate 1/3 by about 1dB for the same FER. In addition to the loss observed by the IRAA-PEG Root-Check LDPC codes, we have observed an incredible feasibility - WHAT DO YOU MEAN BY THAT? THIS EXPRESSION MUST BE REPHRASED. to generate rate-compatible codes.

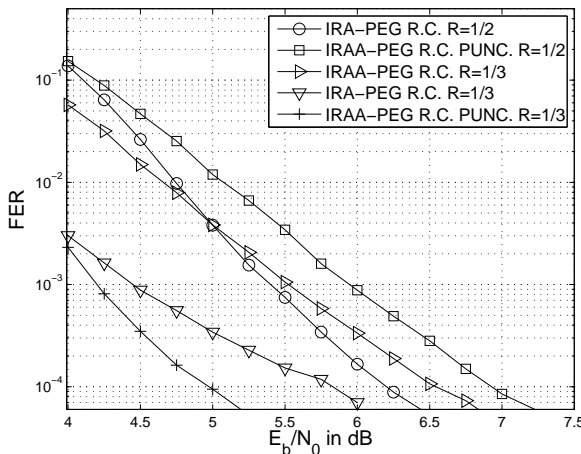


Figure 5. FER performance for IRA-PEG Root-Check and IRAA-PEG Root-Check codes with different code rates over a fast fading channel.

VI. CONCLUSION

A novel PEG-based algorithm has been proposed to design IRA(A)-PEG Root-Check LDPC codes for $F = 2, 3$ fading blocks. Based on simulations, the proposed method was compared to previous works in [6]–[8]. The results demonstrate that the IRA(A)-PEG Root-Check LDPC codes generated by our proposed algorithm perform as well as the QC-PEG Root-Check LDPC codes [8] in a wide range of SNR values and for fast fading channel it can save up to 1dB. In addition, all designed Root-Check LDPC codes presented in this paper are new in the essence that no works were found with respect to IRA Root-Check design style. Furthermore, we would like to reinforce that IRA encoding it is much simpler than existing methods. Moreover, the proposed IRAA code design brings flexibility in terms of rate and coding gains under fast fading channels. There are some ongoing works in this are: first, we are currently looking into the design of Root-Check codes with accumulate repeat-accumulate (ARA) LDPC codes; second, we are investigating the impact toward fast fading channels when the mother code is a Root-Check LDPC code rate 1/2 and by puncturing techniques produce high rate codes; third, we are considering improved decoding strategies [13] for the above mentioned designs.

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